

IEC-GATE-2013 PAPER

Q. No. 1 – 25 Carry One Mark Each

1. A bulb in a staircase has two switches, one switch being at the ground floor and the other one at the first floor. The bulb can be turned ON and also can be turned OFF by any one of the switches irrespective of the state of the other switch. The logic of switching of the bulb resembles
 (A) an AND gate (B) an OR gate (C) an XOR gate (D) a NAND gate

Answer: (C)

Exp: (C)

Let Switches = P_1, P_2

P_1	P_2	Z(o / p)
OFF	OFF	OFF
OFF	ON	ON
ON	OFF	ON
ON	ON	OFF

From Truth Table, it can be verified that Ex-OR logic is implemented.

2. Consider a vector field $\vec{A}(\vec{r})$. The closed loop line integral $\oint \vec{A} \cdot d\vec{l}$ can be expressed as
 (A) $\oiint (\nabla \times \vec{A}) \cdot d\vec{s}$ over the closed surface bounded by the loop
 (B) $\iiint (\nabla \cdot \vec{A}) dv$ over the closed volume bounded by the loop
 (C) $\iiint (\nabla \cdot \vec{A}) dv$ over the open volume bounded by the loop
 (D) $\iint (\nabla \times \vec{A}) \cdot d\vec{s}$ over the closed surface bounded by the loop

Answer: (D)

Exp: (D)

Stoke's Theorem: " The Line Integral of a vector \vec{A} around a closed path L is equal to the integral of curl of \vec{A} over the open surface S enclosed by the closed path L".

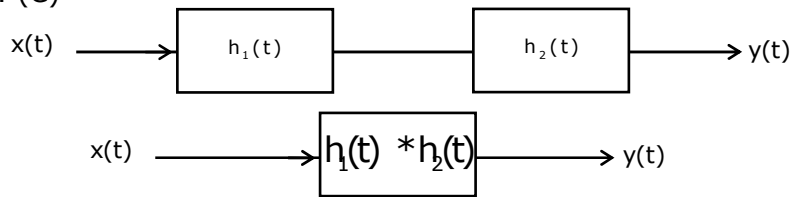
$$\therefore \oint \vec{A} \cdot d\vec{l} = \iint (\nabla \times \vec{A}) \cdot d\vec{s}$$

3. Two systems with impulse responses $h_1(t)$ and $h_2(t)$ are connected in cascade. Then the overall impulse response of the cascaded system is given by
 (A) Product of $h_1(t)$ and $h_2(t)$
 (B) Sum of $h_1(t)$ and $h_2(t)$
 (C) Convolution of $h_1(t)$ and $h_2(t)$
 (D) Subtraction of $h_2(t)$ from $h_1(t)$

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Answer: (C)

Exp:



4. In a forward biased pn junction diode, the sequence of events that best describes the mechanism of current flow is

- (A) injection, and subsequent diffusion and recombination of minority carriers
- (B) injection, and subsequent drift and generation of minority carriers
- (C) extraction, and subsequent diffusion and generation of minority carriers
- (D) extraction, and subsequent drift and recombination of minority carriers

Answer: (A)

5. In IC technology, dry oxidation (using dry oxygen) as compared to wet oxidation (using steam or water vapor) produces

- (A) superior quality oxide with a higher growth rate
- (B) inferior quality oxide with a higher growth rate
- (C) inferior quality oxide with a lower growth rate
- (D) superior quality oxide with a lower growth rate

Answer: (D)

6. The maximum value of θ until which the approximation $\sin \theta \approx \theta$ holds to within 10% error is

- (A) 10°
- (B) 18°
- (C) 50°
- (D) 90°

Answer: (B)

Exp: $\sin \theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \dots$

$$\sin \theta = \theta + \text{error}$$

Error should 10% of θ

$$\text{if } \frac{\theta^3}{6} \leq 0.1 \theta$$

if $\frac{\theta^3}{6} \leq 0.1 \theta$ Then higher order terms also going to be less than 0.1θ

So, $\sin \theta \approx \theta$ approximation is valid

$$\frac{\theta^3}{6} < 0.1 \theta$$

$$\theta^2 < 0.6$$

$$\theta < \sqrt{0.6}$$

Consider $\theta_{\max} = \sqrt{0.6} = 0.7746$ radians

$$\theta(\text{in degrees}) = 0.7746 \times \frac{180}{\pi} \approx 45^\circ$$

Out of all possible options we need to go for max theta below 45° Hence it is 18°

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7. The divergence of the vector field $\vec{A} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$ is
 (A) 0 (B) 1/3 (C) 1 (D) 3

Answer: (D)

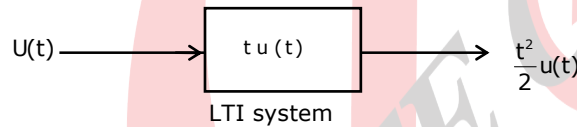
Exp: Given $\vec{A} = x\bar{a}_x + y\bar{a}_y + z\bar{a}_z$
 $\nabla \cdot \vec{A} = \frac{\partial}{\partial x}(A_x) + \frac{\partial}{\partial y}(A_y) + \frac{\partial}{\partial z}(A_z)$
 $A_x = x, A_y = y, A_z = z,$
 $= \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(z)$
 $= 1 + 1 + 1 = 3$

8. The impulse response of a system is $h(t) = tu(t)$. For an input $u(t-1)$, the output is

- (A) $\frac{t^2}{2}u(t)$ (B) $\frac{t(t-1)}{2}u(t-1)$ (C) $\frac{(t-1)^2}{2}u(t-1)$ (D) $\frac{t^2-1}{2}u(t-1)$

Answer: (C)

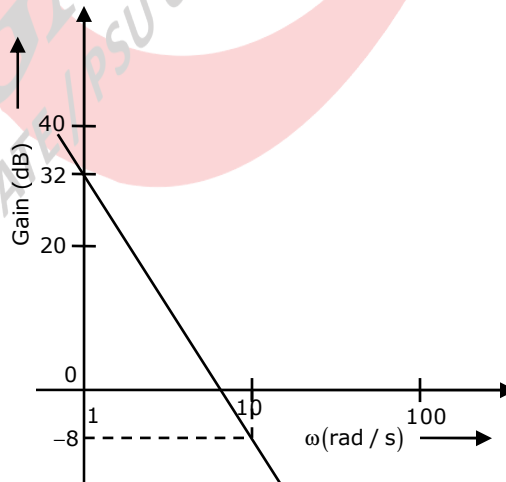
Exp:



For LTI system, if input gets delayed by one unit, output will also get delayed by one unit.

$$u(t-1) \rightarrow \frac{(t-1)^2}{2} u(t-1)$$

9. The Bode plot of a transfer function $G(s)$ is shown in the figure below



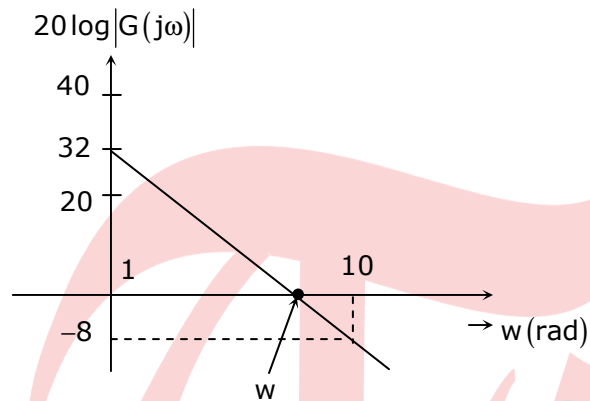
The gain $(20 \log|G(s)|)$ is 32 dB and -8 dB at 1 rad/s and 10 rad/s respectively.

The phase is negative for all ω . The $G(s)$ is

- (A) $\frac{39.8}{s}$ (B) $\frac{39.8}{s^2}$ (C) $\frac{32}{s}$ (D) $\frac{32}{s^2}$

Answer: (B)

Exp: Any two points on same line segment of Bode plot satisfies the equation of straight line.



i.e, $\frac{G_2 - G_1}{\log \omega_2 - \log \omega_1}$ = slope of the line segment.

For the initial straight line

$$\Rightarrow \frac{G_2 - G_1}{\log \omega_2 - \log \omega_1} = -40 \text{ dB/dec}$$

$$\Rightarrow 0 - 32 = -40 \log \left(\frac{\omega}{1} \right)$$

$$\Rightarrow \omega = 6.309 = k^{1/N} ; \text{Where } N \text{ is type of system here initial slope is } -40 \text{ dB/dec} \text{ Hence } N = 2$$

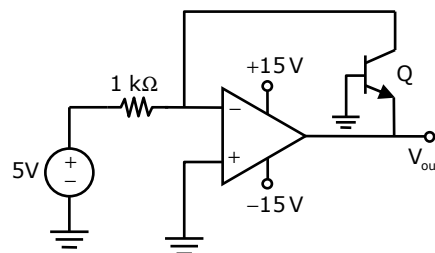
$$\Rightarrow 6.309 = k^{1/2}$$

$$\Rightarrow k = (6.309)^2$$

$$k = 39.8$$

Hence $G(s) = \frac{39.8}{s^2}$

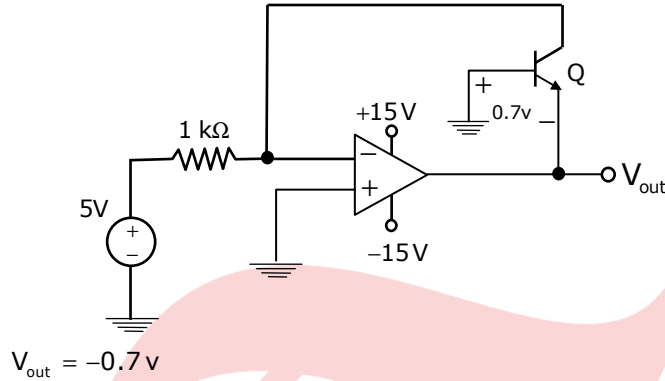
10. In the circuit shown below what is the output voltage (V_{out}) if a silicon transistor Q and an ideal op-amp are used?



- (A) -15V (B) -0.7V (C) +0.7V (D) +15V

Answer: (B)

Exp:



11. Consider a delta connection of resistors and its equivalent star connection as shown below. If all elements of the delta connection are scaled by a factor k , $k > 0$, the elements of the corresponding star equivalent will be scaled by a factor of



- (A) k^2 (B) k (C) $1/k$ (D) \sqrt{k}

Answer: (B)

Exp:

$$R_C = \frac{R_a R_b}{R_a + R_b + R_c}$$

$$R_B = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_A = \frac{R_b R_c}{R_a + R_b + R_c}$$

Above expression shown that if R_a, R_b & R_c is scaled by k , R_A, R_B & R_C is scaled by k only.

12. For 8085 microprocessor, the following program is executed

```
MVI A, 05H;
MVIB, 05H;
PTR: ADD B;
DCR B;
JNZ PTR;
ADI 03H;
HLT;
```

At the end of program, accumulator contains

- (A) 17 H (B) 20 H (C) 23 H (D) 05 H

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Answer: (A)

Exp: Accumulator changes as follows (05 + 05 + 04 +03 +02 +01)H
 At the end of Loop accumulator contains = 14H
 ADI 03H →A=(14+03)=17H

13. The bit rate of a digital communication system is R kbits/s. The modulation used is 32-QAM. The minimum bandwidth required for ISI free transmission is
 (A) $R/10$ Hz (B) $R/10$ kHz (C) $R/5$ Hz (D) $R/5$ kHz

Answer: (B)

Exp: Bit rate given = R Kbits/second
 Modulation = 32-QAM
 No. of bits/symbol = $5 [\log_2 32]$
 Symbol rate = $\frac{R}{5}$ k symbols / second

Finally we are transmitting symbols.

B_T → transmission bandwidth

$$B_T = \frac{R(\text{symbol rate})}{(1 + \alpha)}$$

$$B_T = \frac{R}{5(1 + \alpha)}$$

For B_T to be minimum, α has to be maximum

$$\Rightarrow B_T = \frac{R}{5 \times 2} = \frac{R}{10}$$

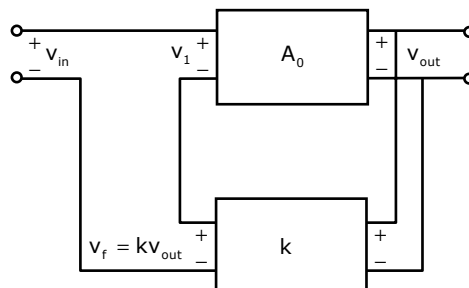
Maximum value of α is '1' which is a roll off factor

14. For a periodic signal $v(t) = 30 \sin 100t + 10 \cos 300t + 6 \sin(500t + \pi/4)$, the fundamental frequency in rad/s
 (A) 100 (B) 300 (C) 500 (D) 1500

Answer: (A)

Exp: $\omega_0 = 100$ rad / sec fundamental
 $3\omega_0 = 300$ rad / sec third harmonic
 $5\omega_0 = 500$ rad / sec fifth harmonic

15. In a voltage-voltage feedback as shown below, which one of the following statements is TRUE if the gain k is increased?

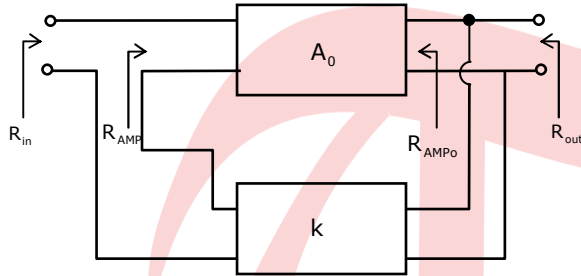


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- (A) The input impedance increases and output impedance decreases
- (B) The input impedance increases and output impedance also increases
- (C) The input impedance decreases and output impedance also decreases
- (D) The input impedance decreases and output impedance increases

Answer: (A)

Exp: In voltage-voltage feedback



$$R_{in} = R_{AMP} (1 + A_0 K)$$

$$R_{out} = \frac{R_{AMPo}}{1 + A_0 K}$$

as $K \uparrow$

$$R_{in} \uparrow, \quad R_{out} \downarrow$$

16. A band-limited signal with a maximum frequency of 5 kHz is to be sampled. According to the sampling theorem, the sampling frequency which is not valid is
- (A) 5 kHz
 - (B) 12 kHz
 - (C) 15 kHz
 - (D) 20 kHz

Answer: (A)

Exp: Given: $f_m = 5\text{kHz}$

According to sampling frequency

$$f_s \geq 2f_m$$

$$f_s \geq 10 \text{ kHz}$$

So, only in option (a) it is less than 10KHz ie., (5KHz)

17. In a MOSFET operating in the saturation region, the channel length modulation effect causes
- (A) an increase in the gate-source capacitance
 - (B) a decrease in the Transconductance
 - (C) a decrease in the unity-gain cutoff frequency
 - (D) a decrease in the output resistance

Answer: (D)

Exp: No channel length modulation

$$I_{DS} = \frac{1}{2} \times k (V_{GS} - V_T)^2$$

$$\frac{\partial I_{DS}}{\partial V_{DS}} = \frac{1}{r_{ds}} \Rightarrow \frac{1}{r_{ds}} = 0 \Rightarrow r_{ds} = \infty$$

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under the presence of channel length modulation

$$\frac{\partial I_{DS}}{\partial V_{DS}} = \lambda I_{Dsat} = \frac{1}{r_0}$$

$$r_0 = \frac{1}{\lambda I_{Dsat}} \quad \therefore \text{which is reduced from } \infty \text{ to finite value}$$

18. Which one of the following statements is NOT TRUE for a continuous time causal and stable LTI system?
- (A) All the poles of the system must lie on the left side of the $j\omega$ axis
- (B) Zeros of the system can lie anywhere in the s -plane
- (C) All the poles must lie within $|s| = 1$
- (D) All the roots of the characteristic equation must be located on the left side of the $j\omega$ axis

Answer: (C)

Exp: For an LTI system to be stable and causal all poles or roots of characteristic equation must lie on LHS of s -plane i.e., left hand side of $j\omega$ -axis
[Refer Laplace transform].

19. The minimum Eigen value of the following matrix is

$$\begin{bmatrix} 3 & 5 & 2 \\ 5 & 12 & 7 \\ 2 & 7 & 5 \end{bmatrix}$$

- (A) 0 (B) 1 (C) 2 (D) 3

Answer: (A)

Exp: $\begin{bmatrix} 3 & 5 & 2 \\ 5 & 12 & 7 \\ 2 & 7 & 5 \end{bmatrix} \xrightarrow{C_1 \leftrightarrow C_2} \begin{bmatrix} 5 & 5 & 2 \\ 12 & 12 & 7 \\ 7 & 7 & 5 \end{bmatrix} \Rightarrow \text{determinant} = 0,$

So the matrix is singular

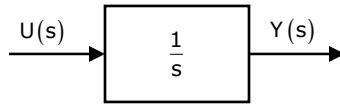
Therefore atleast one of the Eigen value is '0'

As the choices are non negative, the minimum Eigen value is '0'

20. A polynomial $f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x - a_0$ with all coefficients positive has
- (A) no real roots
- (B) no negative real root
- (C) odd number of real roots
- (D) at least one positive and one negative real root

Answer: (D) Use Routh Hurwitz Criteria to get the condition.

21. Assuming zero initial condition, the response $y(t)$ of the system given below to a unit step input $u(t)$ is

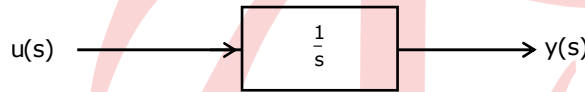


- (A) $u(t)$ (B) $tu(t)$ (C) $\frac{t^2}{2}u(t)$ (D) $e^{-t}u(t)$

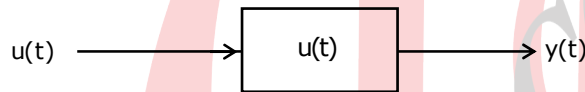
Answer: (B)

Exp: (B)

Integration of unit step function is ramp output

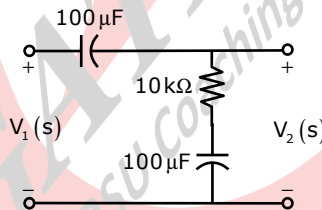


Writing in time domain



$$y(t) = u(t) * u(t) = tu(t)$$

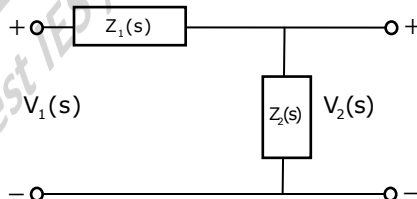
22. The transfer function $\frac{V_2(s)}{V_1(s)}$ of the circuit shown below is



- (A) $\frac{0.5s + 1}{s + 1}$ (B) $\frac{3s + 6}{s + 2}$ (C) $\frac{s + 2}{s + 1}$ (D) $\frac{s + 1}{s + 2}$

Answer: (D)

Exp: (D)



$$z_1(s) = \frac{1}{10^{-4}s}, \quad z_2(s) = \frac{s+1}{10^{-4}s}$$

$$\frac{V_2(s)}{V_1(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)} = \frac{s+1}{s+2}$$

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23. A source $v_s(t) = V \cos 100\pi t$ has an internal impedance of $(4 + j3)\Omega$. If a purely resistive load connected to this source has to extract the maximum power out of the source, its value in Ω should be

(A) 3 (B) 4 (C) 5 (D) 7

Answer: (C)

Exp: For maximum power Transfer

$$\begin{aligned} R_L &= |Z_s| \\ &= \sqrt{4^2 + 3^2} \\ &= 5\Omega \end{aligned}$$

24. The return loss of a device is found to be 20 dB. The voltage standing wave ratio (VSWR) and magnitude of reflection coefficient are respectively

(A) 1.22 and 0.1 (B) 0.81 and 0.1 (C) -1.22 and 0.1 (D) 2.44 and 0.2

Answer: (A)

Exp: The reflection co-efficient is $-20 \log \Gamma = 20\text{dB}$

$$\Rightarrow \log \Gamma = -1\text{dB}$$

$$\Rightarrow \Gamma = 10^{-1} \Rightarrow \Gamma = 0.1$$

Relation between Γ and VSWR is

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$S = \frac{1 + 0.1}{1 - 0.1} = \frac{1.1}{0.9}$$

$$s = 1.22$$

25. Let $g(t) = e^{-\pi t^2}$, and $h(t)$ is a filter matched to $g(t)$. If $g(t)$ is applied as input to $h(t)$, then the Fourier transform of the output is

(A) $e^{-\pi f^2}$ (B) $e^{-\pi f^2/2}$ (C) $e^{-\pi|f|}$ (D) $e^{-2\pi f^2}$

Answer: (D)

Exp:

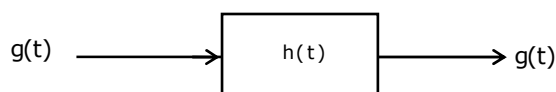
The concept of matched filter assumes that the input signal is of the same form $g(t)$ as the transmitted signal (except difference in amplitude). This requires that the shape of the transmitted signal not change on reflection.

$$h(t) = g(-t) \Leftrightarrow H(f) = G^*(f)$$

$$G^*(f) = G(f) \quad \therefore G(f) \text{ is real}$$

$$g(t) = e^{-\pi t^2} \Leftrightarrow e^{-\pi f^2} \text{ (fourier transform)}$$

$$\Rightarrow y(t) \Leftrightarrow e^{-\pi f^2} \times e^{-\pi f^2} = e^{-2\pi f^2}$$



$$y(t) = h(t) * g(t) \text{ [convolution]}$$

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26. Let U and V be two independent zero mean Gaussian random variables of variances $\frac{1}{4}$ and $\frac{1}{9}$ respectively. The probability $P(3V \geq 2U)$ is

- (A) $\frac{4}{9}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{5}{9}$

Answer: (B)

Exp: $p(3V \geq 2U) = p(3V - 2 \geq 0) = p(W \geq 0), W = 3V - 2U$

U, V are independent random variables and $U \sim N\left(0, \frac{1}{4}\right)$

$V \sim N\left(0, \frac{1}{9}\right)$

$\therefore W = 3V - 2U \sim N\left(0, 9 \times \frac{1}{4} + 4 \times \frac{1}{9}\right)$

$W \sim N(0, 2)$ i.e., W has mean $\mu = 0$ and variance, $\sigma^2 = 2$

$\therefore p(W \geq 0) = p\left(\frac{W - \mu}{\sigma} \geq \frac{0 - \mu}{\sigma}\right)$

$= p(Z \geq 0)$, Z is standard normal variants

$= 0.5 = \frac{1}{2}$

27. Let A be an $m \times n$ matrix and B an $n \times m$ matrix. It is given that determinant $(I_m + AB) = \text{determinant}(I_n + BA)$, where I_k is the $k \times k$ identity matrix. Using the above property, the determinant of the matrix given below is

$$\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

- (A) 2 (B) 5 (C) 8 (D) 16

Answer: (B)

Exp: Let $A = [1 \ 1 \ 1 \ 1]_{1 \times 4}$; $B = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}_{4 \times 1}$; $I_1 = 1$; $I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

then $AB = [4]$; $BA = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ Here $m = 1$; $n = 4$

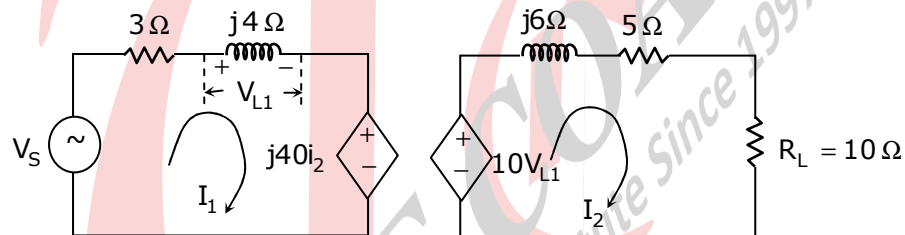
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and $\det(I_1 + AB) = \det(I_4 + BA)$

$$\Rightarrow \det \text{ of } [5] = \det \text{ of } \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

$$\therefore \det \text{ of } \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

28. In the circuit shown below, if the source voltage $V_s = 100 \angle 53.13^\circ \text{V}$ then the Thevenin's equivalent voltage in Volts as seen by the load resistance R_L is



- (A) $100 \angle 90^\circ$ (B) $800 \angle 0^\circ$ (C) $800 \angle 90^\circ$ (D) $100 \angle 60^\circ$

Answer: (C)

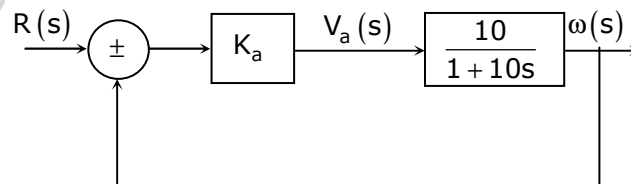
Exp: $V_{TH} = 10V_{L1}$

$$V_{L1} = \frac{V_c}{3 + j4} = \frac{100 \angle 53.13}{5} - \tan^{-1}\left(\frac{4}{3}\right) \times j4$$

$$V_{L1} = 80 \angle 90^\circ$$

$$V_{TH} = 800 \angle 90^\circ$$

29. The open-loop transfer function of a dc motor is given as $\frac{\omega(s)}{V_a(s)} = \frac{10}{1 + 10s}$, when connected in feedback as shown below, the approximate value of K_a that will reduce the time constant of the closed loop system by one hundred times as compared to that of the open-loop system is



- (A) 1 (B) 5 (C) 10 (D) 100

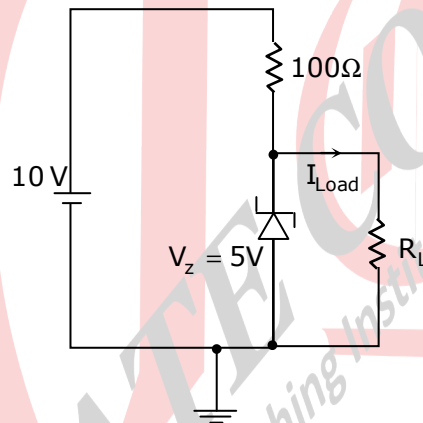
Answer: (C)

Exp: $\tau_{\text{open loop}} = 10$

$$\tau_{\text{closed loop}} = \frac{10}{100} = \frac{10}{1 + 10K_a} \Rightarrow K_a = 9.9 \approx 10.$$

$$\begin{aligned} \text{CLTF} &= \frac{10k}{1 + \frac{10K_a}{1 + 10s}} = \frac{10K_a}{1 + \frac{10s}{1 + 10K_a} (1 + 10K_a)} \\ &= 10 \end{aligned}$$

30. In the circuit shown below, the knee current of the ideal Zener diode is 10mA. To maintain 5V across R_L , the minimum value of R_L in Ω and the minimum power rating of the Zener diode in mW, respectively, are



- (A) 125 and 125 (B) 125 and 250
(C) 250 and 125 (D) 250 and 250

Answer: (B)

Exp $R_{L\text{min}} = \frac{5}{I_{L\text{max}}}$

$$I_{100} = \frac{10-5}{100} = \frac{5}{100} = 50 \text{ mA}$$

$$I_{L\text{max}} = I_{100} - I_{\text{knee}} = 40 \text{ mA}$$

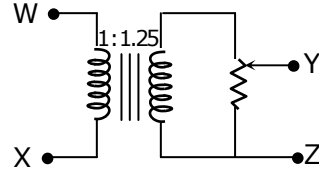
$$R_{L\text{min}} = \frac{5}{40} \times 1000 = 125\Omega$$

Minimum power rating of Zener should $= 50 \text{ mA} \times 5V$
 $= 250 \text{ mW}$

31. The following arrangement consists of an ideal transformer and an attenuator which attenuates by a factor of 0.8 An ac voltage $V_{\text{WX1}} = 100V$ is applied across WX to get an open circuit voltage V_{YZ1} across YZ. Next, an ac voltage $V_{\text{YZ2}} = 100V$ is applied across YZ to get an open circuit voltage V_{WX2} across WX. Then, $V_{\text{YZ1}} / V_{\text{WX1}}$, $V_{\text{WX2}} / V_{\text{YZ2}}$ are respectively.

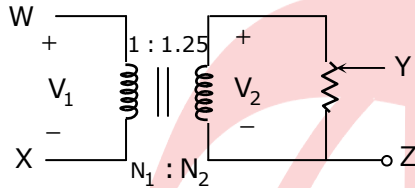
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- (A) 125 / 100 and 80 / 100
- (B) 100 / 100 and 80 / 100
- (C) 100 / 100 and 100 / 100
- (D) 80 / 100 and 80 / 100



Answer: (C)

Exp:



For a transform

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

$$\Rightarrow V_2 = 1.125 \times V_1$$

The potentiometer gives an attenuation factor of 0.8 over v_2

$$\text{Hence } V_{yz} = 0.8v_2 \Rightarrow \frac{V_{yz}}{0.8} = v_2$$

$$\Rightarrow \frac{V_{yz}}{0.8} = 1.125 \times V_{wx}$$

$$\Rightarrow V_{yz} = V_{wx} \Rightarrow \frac{V_{yz1}}{V_{wx1}} = \frac{100}{100}$$

Since potentiometer and transformer are bilateral elements. Hence $\frac{V_{wx2}}{V_{yz1}} = \frac{100}{100}$

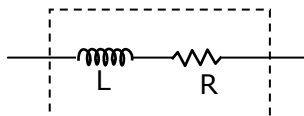
32. Two magnetically uncoupled inductive coils have Q factors q_1 and q_2 at the chosen operating frequency. Their respective resistances are R_1 and R_2 . When connected in series, their effective Q factor at the same operating frequency is

- (A) $q_1 + q_2$
- (B) $(1 / q_1) + (1 / q_2)$
- (C) $(q_1 R_1 + q_2 R_2) / (R_1 + R_2)$
- (D) $(q_1 R_2 + q_2 R_1) / (R_1 + R_2)$

Answer: (C)

Exp: Q Factor of a inductive coil.

$$\bar{Q} = \frac{wL}{R} \Rightarrow Q_1 = \frac{wL_1}{R_1} \text{ \& } Q_2 = \frac{wL_2}{R_2}$$



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When such two coils are connected in series individual inductances and resistances are added.

$$\text{Hence, } L_{\text{eq}} = L_1 + L_2$$

$$R_{\text{eq}} = R_1 + R_2$$

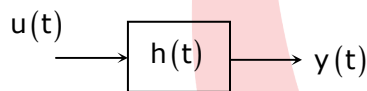
$$\text{Hence } Q_{\text{eq}} = \frac{\omega L_{\text{eq}}}{R_{\text{eq}}} = \frac{\omega(L_1 + L_2)}{(R_1 + R_2)} = \frac{\frac{\omega L_1}{R_1} + \frac{\omega L_2}{R_2}}{\frac{R_1}{R_1 R_2} + \frac{R_2}{R_1 R_2}}$$

$$= \frac{\frac{Q_1}{R_2} + \frac{Q_2}{R_1}}{\frac{1}{R_2} + \frac{1}{R_1}} = \frac{Q_1 R_1 + Q_2 R_2}{R_1 + R_2}$$

33. The impulse response of a continuous time system is given by $h(t) = \delta(t - 1) + \delta(t - 3)$. The value of the step response at $t = 2$ is
 (A) 0 (B) 1 (C) 2 (D) 3

Answer: (B)

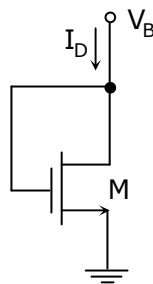
Exp: $h(t) = \delta(t - 1) + \delta(t - 3)$



$$y(t) = u(t - 1) + u(t - 3)$$

$$y(2) = u(1) + u(-1) = 1$$

34. The small-signal resistance (i.e., dV_B / dI_D) in $k\Omega$ offered by the n-channel MOSFET M shown in the figure below, at a bias point of $V_B = 2\text{ V}$ is (device data for M: device Transconductance parameter $k_N = \mu_n C'_{\text{ox}} (W / L) = 40\mu\text{A} / \text{V}^2$, threshold voltage $V_{\text{TN}} = 1\text{ V}$, and neglect body effect and channel length modulation effects)



- (A) 12.5 (B) 25 (C) 50 (D) 100

Answer: (B)

Exp:

$$\frac{dV_B}{dI_D} = ?$$

$$V_B = V_{DS} = V_{GS}$$

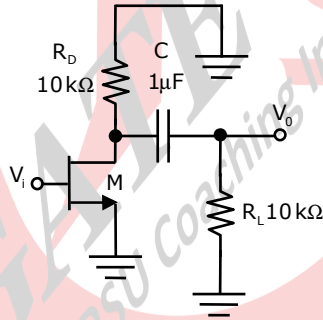
∴ M is in saturation

$$I_D = \frac{1}{2} \times 40 \times 10^{-6} (V_{DS} - V_T)^2$$

$$\frac{\partial I_D}{\partial V_{DS}} = 40 \times 10^{-6} (2 - 1) = 40 \times 10^{-6}$$

$$\frac{\partial V_{DS}}{\partial I_D} = \frac{\partial V_B}{\partial I_D} = 25k\Omega$$

35. The ac schematic of an NMOS common-source stage is shown in the figure below, where part of the biasing circuits has been omitted for simplicity. For the n-channel MOSFET M, the Transconductance $g_m = 1\text{mA/V}$, and body effect and channel length modulation effect are to be neglected. The lower cutoff frequency in Hz of the circuit is approximately at



(A) 8

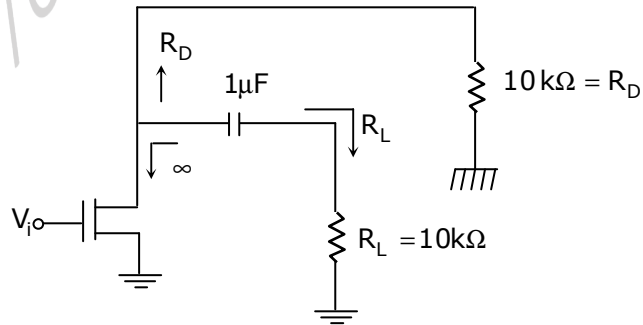
(B) 32

(C) 50

(D) 200

Answer: (A)

Exp:



$$f_{cut} = \frac{1}{2\pi(R_D + R_L)C}$$

$$= 8\text{Hz}$$

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36. A system is described by the differential equation $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y(t) = x(t)$.

Let $x(t)$ be a rectangular pulse given by

$$x(t) = \begin{cases} 1 & 0 < t < 2 \\ 0 & \text{otherwise} \end{cases}$$

Assuming that $y(0)=0$ and $\frac{dy}{dt} = 0$ at $t = 0$, the Laplace transform of $y(t)$ is

- (A) $\frac{e^{-2s}}{s(s+2)(s+3)}$ (B) $\frac{1 - e^{-2s}}{s(s+2)(s+3)}$
 (C) $\frac{e^{-2s}}{(s+2)(s+3)}$ (D) $\frac{1 - e^{-2s}}{(s+2)(s+3)}$

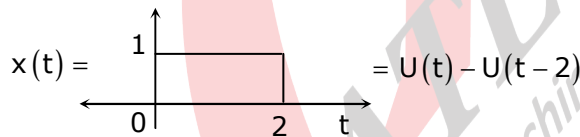
Answer: (B)

Exp: Writing in terms of laplace transform.

$$S^2y(s) + 5sy(s) + 6y(s) = x(s)$$

$$\Rightarrow y(s) = \frac{x(s)}{s^2 + 5s + 6}$$

$$x(s) = \left(\frac{1 - e^{-2s}}{s} \right)$$



⇕

$$x(s) = \frac{1 - e^{-2s}}{s}$$

$$\Rightarrow Y(s) = \frac{1 - e^{-2s}}{s(s+2)(s+3)}$$

37. A system described by a linear, constant coefficient, ordinary, first order differential equation has an exact solution given by $y(t)$ for $t > 0$, when the forcing function is $x(t)$ and the initial condition is $y(0)$. If one wishes to modify the system so that the solution becomes $-2y(t)$ for $t > 0$, we need to

- (A) change the initial condition to $-y(0)$ and the forcing function to $2x(t)$
 (B) change the initial condition to $2y(0)$ and the forcing function to $-x(t)$
 (C) change the initial condition to $j\sqrt{2}y(0)$ and the forcing function to $j\sqrt{2}x(t)$
 (D) change the initial condition to $-2y(0)$ and the forcing function to $-2x(t)$

Answer: (D)

Exp: $\frac{dy(t)}{dt} + ky(t) = x(t)$

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$$sY(s) - y(0) + kY(s) = X(s)$$

$$Y(s)[s+k] = X(s) - Y(0)$$

$$Y(s) = \frac{X(s) - Y(0)}{s+k}$$

$$Y(s) = \frac{X(s)}{s+k} - \frac{Y(0)}{s+k}$$

$$y(t) = e^{-kt}x(t) - y(0)e^{-kt}$$

So if we want $-2y(t)$ as a solution both $x(t)$ and $y(0)$ has to be doubled and multiplied by -ve sign

$$x(t) \rightarrow -2x(t)$$

$$y(0) \rightarrow -2y(0)$$

38. Consider two identically distributed zero-mean random variables U and V. Let the cumulative distribution functions of U and 2V be F(x) and G(x) respectively. Then, for all values of x

(A) $F(x) - G(x) \leq 0$

(B) $F(x) - G(x) \geq 0$

(C) $(F(x) - G(x)) \cdot x \leq 0$

(D) $(F(x) - G(x)) \cdot x \geq 0$

Answer: (D)

Exp: $F(x) = P\{X \leq x\}$

$$G(x) = P\{2X \leq x\}$$

$$= P\{X \leq \frac{x}{2}\}$$

For positive value of x,

$F(x) - G(x)$ is always greater than zero

For negative value of x.

$F(x) - G(x)$ is -ve

but $(F(x) - G(x)) \cdot x \geq 0$

39. The DFT of vector $[a \ b \ c \ d]$ is the vector $[\alpha \ \beta \ \gamma \ \delta]$. Consider the product

$$[p \ q \ r \ s] = [a \ b \ c \ d] \begin{bmatrix} a & b & c & d \\ d & a & b & c \\ c & d & a & b \\ b & c & d & a \end{bmatrix}$$

The DFT of the vector $[p \ q \ r \ s]$ is a scaled version of

(A) $[\alpha^2 \ \beta^2 \ \gamma^2 \ \delta^2]$

(B) $[\sqrt{\alpha} \ \sqrt{\beta} \ \sqrt{\gamma} \ \sqrt{\delta}]$

(C) $[\alpha + \beta \ \beta + \delta \ \delta + \gamma \ \gamma + \alpha]$

(D) $[\alpha \ \beta \ \gamma \ \delta]$

Answer: (A)

Exp: $x(n) = [a \ b \ c \ d]^T$

$$x(n) \otimes_{4 \times 4} x(n) = \begin{bmatrix} a & d & c & d \\ b & a & d & c \\ c & b & a & d \\ d & c & b & a \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}_{4 \times 1}$$

$$x(n) \otimes^4 x(n) = [a \ b \ c \ d] \begin{bmatrix} a & b & c & d \\ d & a & b & c \\ c & d & a & b \\ b & c & d & a \end{bmatrix}$$

$$[p \ q \ r \ s] = x(n) \otimes x(n)$$

$$x(n) \xrightarrow{\text{DFT}} x(k) [\alpha \ \beta \ \gamma \ \delta]$$

$$\text{DFT}\{p \ q \ r \ s\} = x(k)x(k) = x^2(ck) = [\alpha^2 \ \beta^2 \ \gamma^2 \ \delta^2]$$

40. The signal flow graph for a system is given below. The transfer function $\frac{Y(s)}{U(s)}$ for

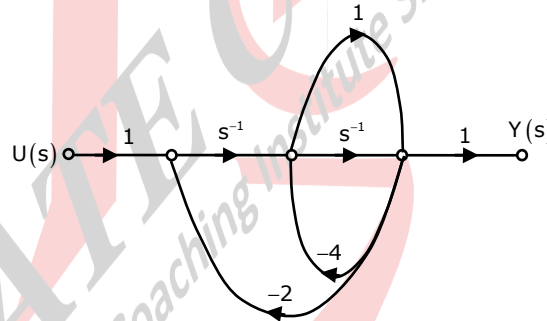
this system is

(A) $\frac{s+1}{5s^2+6s+2}$

(B) $\frac{s+1}{s^2+6s+2}$

(C) $\frac{s+1}{s^2+4s+2}$

(D) $\frac{1}{5s^2+6s+2}$



Answer: (A)

Exp: By using Mason's gain formula

$$\frac{y(s)}{u(s)} = \frac{s^{-2} + s^{-1}}{1 - [-2s^{-2} - 4s^{-1} - 2s^{-1} - 4] + 0} = \frac{s^{-2}[s+1]}{2s^2 + 6s^{-1} + 5} = \frac{s+1}{5s^2+6s+2}$$

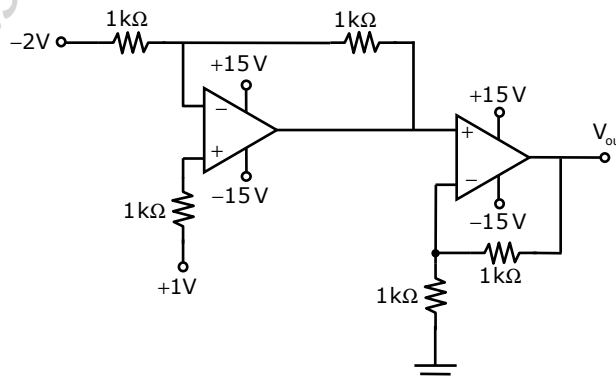
41. In the circuit shown below the op-amps are ideal. The V_{out} in Volts is

(A) 4

(B) 6

(C) 8

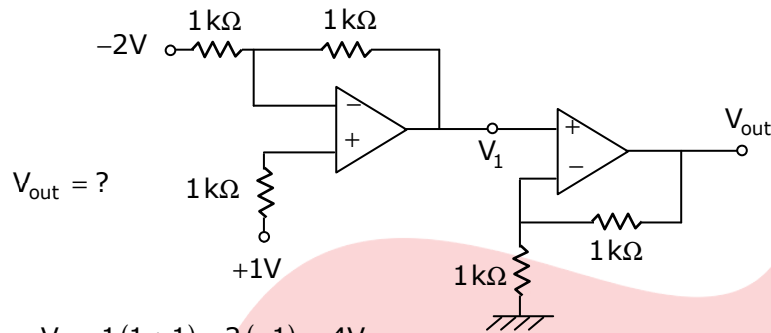
(D) 10



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Answer: (C)

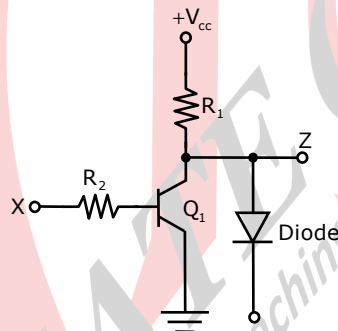
Exp:



$$V_1 = 1(1 + 1) - 2(-1) = 4V$$

$$V_{out} = 2V_1 = 8V$$

42. In the circuit shown below, Q_1 has negligible collector-to-emitter saturation voltage and the diode drops negligible voltage across it under forward bias. If V_{cc} is +5V, X and Y are digital signals with 0 V as logic 0 and V_{cc} as logic 1, then the Boolean expression for Z is



(A) XY

(B) $\bar{X}Y$

(C) $X\bar{Y}$

(D) $\bar{X}\bar{Y}$

Answer: (B)

Exp: (B)

X	Y	Z
0	0	0
0	1	1
1	0	0
1	1	0

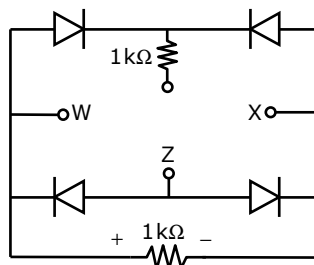
43. A voltage $1000\sin\omega t$ Volts is applied across YZ. Assuming ideal diodes, the voltage measured across WX in Volts, is

(A) $\sin\omega t$

(B) $(\sin\omega t + |\sin\omega t|) / 2$

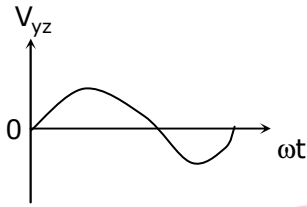
(C) $(\sin\omega t - |\sin\omega t|) / 2$

(D) 0 for all t

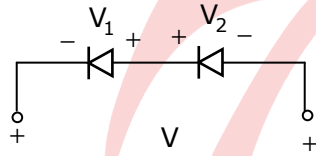


Answer: (D)

Exp:.



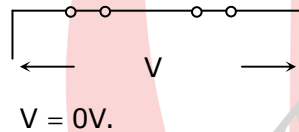
During the half cycle
All Diodes OFF & Hence



$$|V_1| = |V_2|$$

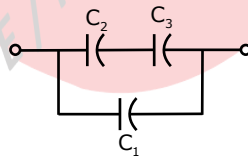
$$V = 0V$$

During -Ve half cycle



44. Three capacitors C_1, C_2 and C_3 whose values are $10\mu F, 5\mu F,$ and $2\mu F$ respectively, have breakdown voltages of $10V, 5V,$ and $2V$ respectively. For the interconnection shown below, the maximum safe voltage in Volts that can be applied across the combination, and the corresponding total charge in μC stored in the effective capacitance across the terminals are respectively.

- (A) 2.8 and 36
- (B) 7 and 119
- (C) 2.8 and 32
- (D) 7 and 80



Answer: (C)

Exp: $\frac{V_{C_3}}{C_2 + C_3} \leq 5V \Rightarrow \frac{2V}{7} \leq 5V \dots\dots\dots(1)$

$$\frac{V_{C_2}}{C_2 + C_3} \leq 2 \Rightarrow \frac{5V}{7} \leq 2V \dots\dots\dots(2)$$

$$V \leq 10V \Rightarrow V \leq 10V \dots\dots\dots(3)$$

From (1), $V \leq 17.5$ Volts

From (2), $V \leq 2.8$ Volts

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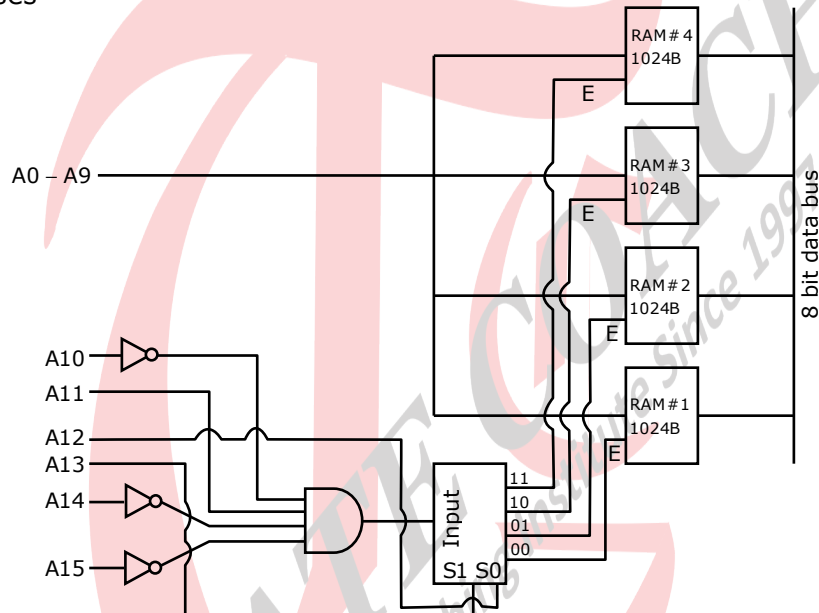
From (3), $V \leq 10$ Volts

To operate Circuit safe, V should be minimum of those $= 2.8V$

$$C_{\text{eff}} = C_1 + (C_2 \parallel C_3) = 10\mu\text{F} + \frac{10}{7}\mu\text{F} = \frac{80}{7}\mu\text{F}$$

$$Q = C_{\text{eff}} \times 2.8V = 32\mu\text{C}$$

45. There are four chips each of 1024 bytes connected to a 16 bit address bus as shown in the figure below. RAMs 1,2,3 and 4 respectively are mapped to addresses



- (A) 0C00H – 0FFFH, 1C00H – 1FFFH, 2C00H – 2FFFH, 3C00H – 3FFFH
 (B) 1800H – 1FFFH, 2800H – 2FFFH, 3800H – 3FFFH, 4800H – 4FFFH
 (C) 0500H – 08FFH, 1500H – 18FFH, 3500H – 38FFH, 5500H – 58FFH
 (D) 0800H – 0BFFH, 1800H – 1BFFH, 2800H – 2BFFH, 3800H – 3BFFH

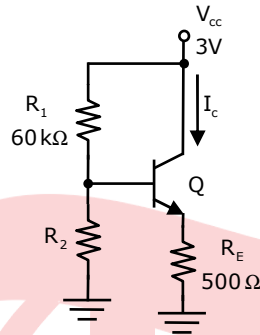
Answer: (D)

Exp: (D)

	A_{14}	$A_{13}(s_1)$	$A_{12}(s_0)$	A_{11}	A_{10}	A_9		A_0
Chip # 1	0	0	0	1	0	0	0=0800H
	0	0	0	1	0	1	0=0BFFH
Chip # 2	0	0	1	1	0	0	0=1800H
	0	0	1	1	0	1	1=1BFFH
Chip # 3	0	1	0	1	0	0	0=2800H
	0	1	0	1	0	1	1=2BFFH
Chip # 4	0	1	1	1	0	0	0=3800H
	0	1	1	1	0	1	0=3BFFH

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46. In the circuit shown below, the silicon npn transistor Q has a very high value of β . The required value of R_2 in $k\Omega$ to produce $I_C = 1\text{ mA}$ is



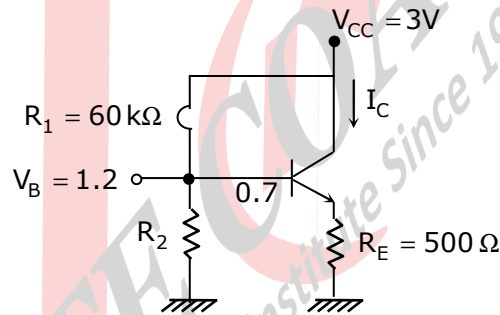
- (A) 20 (B) 30 (C) 40 (D) 50

Answer: (C)

Exp

$$3. \frac{R_2}{60 + R_2} = 1.2$$

$$R_2 = 40\text{ k}\Omega$$



47. Let U and V be two independent and identically distributed random variables such that $P(U = +1) = P(U = -1) = \frac{1}{2}$. The entropy $H(U + V)$ in bits is

- (A) $3/4$ (B) 1 (C) $3/2$ (D) $\log_2 3$

Answer: (C)

Exp:

U	V	(U + V)
+1	+1	+2
+1	-1	0
-1	+1	0
-1	-1	-2

$$P\{U + V = +2\} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$P\{U + V = 0\} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

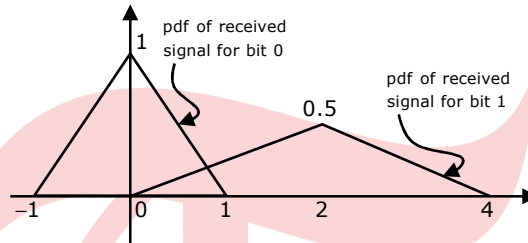
$$P\{U + V = -2\} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$\Rightarrow H\{U + V\} = \frac{1}{2} \log_2 2 + 2 \times \frac{1}{4} \log_2 4$$

$$= \frac{1}{2} + 1 = \frac{3}{2}$$

Common Data Questions: 48 & 49

Bits 1 and 0 are transmitted with equal probability. At the receiver, the pdf of the respective received signals for both bits are as shown below.



48. If the detection threshold is 1, the BER will be

- (A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $\frac{1}{16}$

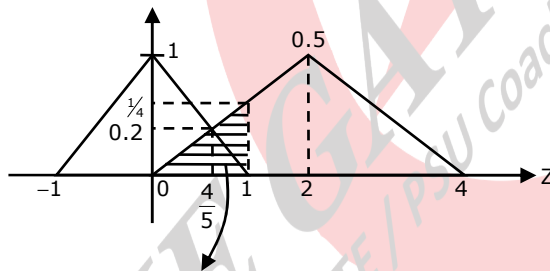
Answer: (D)

Exp: $P_e = P(0)P(1/0) + P(1)P(0/1)$

If Detection threshold = 1

$$P(0) = P(1) = \frac{1}{2}$$

$$P\left(\frac{Y=1}{X=0}\right) = \int_0^1 f\left(\frac{Z}{1}\right) dz$$



$$P(1/0) = \frac{1}{2} \times \frac{1}{4} \times 1 = \frac{1}{8}$$

$$P(0/1) = 0$$

$$P_e = \frac{1}{2} \times \frac{1}{8} + \frac{1}{2} \times 0 = \frac{1}{16}$$

49. The optimum threshold to achieve minimum bit error rate (BER) is

- (A) $\frac{1}{2}$ (B) $\frac{4}{5}$ (C) 1 (D) $\frac{3}{2}$

Answer: (B)

Exp: Optimum threshold = The point of intersection of two pdf's

$$f\left(\frac{z}{0}\right) = 1 - |z| \quad |z| \leq 1$$

$$f\left(\frac{z}{1}\right) = \frac{z}{4} \quad 0 < z < 2$$

The point of intersection which decides optimum threshold

$$\Rightarrow 1 - z = \frac{z}{4}$$

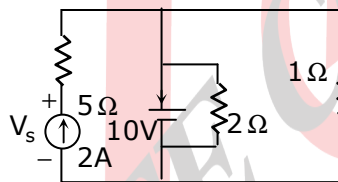
$$1 = \frac{z}{4} + z$$

$$1 = \frac{5z}{4}$$

$$z = \frac{4}{5}$$

Common Data Questions: 50 & 51

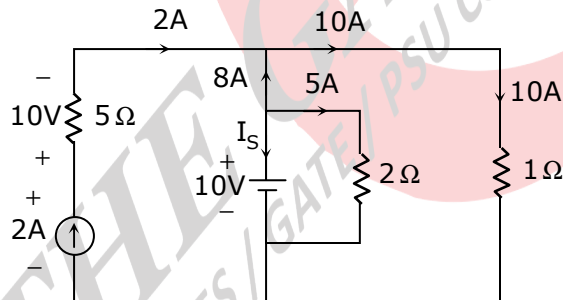
Consider the following figure



50. The current I_S in Amps in the voltage source, and voltage V_S in Volts across the current source respectively, are
 (A) 13, -20 (B) 8, -10 (C) -8, 20 (D) -13, 20

Answer: (D)

Exp:



$$I_S + 8 + 5 = 0$$

$$I_S = -13A$$

51. The current in the 1Ω resistor in Amps is
 (A) 2 (B) 3.33 (C) 10 (D) 12

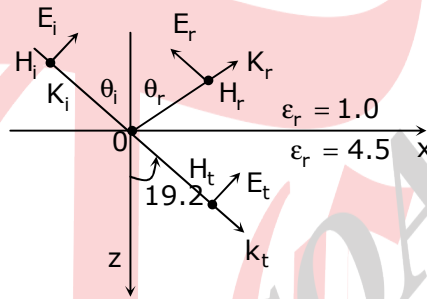
Answer: (C)

Exp: $I_{1\Omega} = 10A$ we can use principle of superposition to determine the current across 1 ohm resistance.

Linked Answer Questions: Q.52 to Q.55 Carry Two Marks Each

Statement for Linked Answer Questions: 52 & 53

A monochromatic plane wave of wavelength $\lambda = 600\mu\text{m}$ is propagating in the direction as shown in the figure below. \vec{E}_i , \vec{E}_r , and \vec{E}_t denote incident, reflected, and transmitted electric field vectors associated with the wave.



52. The angle of incidence θ_1 and the expression for \vec{E}_i are

- (A) 60° and $\frac{E_0}{\sqrt{2}} (\hat{a}_x - \hat{a}_z) e^{-j\frac{\pi \times 10^4 (x+z)}{3\sqrt{2}}}$ V/m
- (B) 45° and $\frac{E_0}{\sqrt{2}} (\hat{a}_x + \hat{a}_z) e^{j\frac{\pi \times 10^4 z}{3}}$ V/m
- (C) 45° and $\frac{E_0}{\sqrt{2}} (\hat{a}_x - \hat{a}_z) e^{-j\frac{\pi \times 10^4 (x+z)}{3\sqrt{2}}}$ V/m
- (D) 65° and $\frac{E_0}{\sqrt{2}} (\hat{a}_x - \hat{a}_z) e^{-j\frac{\pi \times 10^4 z}{3}}$ V/m

Answer: (C)

Exp: (C)

The given oblique incidence is a vertical polarization i.e., \vec{E}_i is parallel to the plane of incidence, \vec{H}_i is perpendicular to the plane of incidence.

$$\vec{E}_i = E_0 [\cos \theta_i \hat{a}_x - \sin \theta_i \hat{a}_z] \times e^{-j\beta_1 [x \sin \theta_i + z \cos \theta_i]} \dots \dots (1)$$

From the given problem

$$n_1 \sin \theta_i = n_2 \sin \theta_t \dots \dots \dots (2)$$

$$\Rightarrow \sqrt{\epsilon_{r1}} \sin \theta_i = \sqrt{\epsilon_{r2}} \sin \theta_t$$

$$\Rightarrow 1 \cdot \sin \theta_i = 2.1213 \times \sin(19.2)$$

$$\sin \theta_i = 0.6976$$

$$\theta_i = \sin^{-1}(0.6976)$$

$$\theta_i \approx 45^\circ$$

∴ the angle of incidence is 45°

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{600 \times 10^{-6}} = \frac{\pi}{3} 10^4 \quad \theta_i = 45, \quad \theta_r = 19.2$$

substituting equation (1) we get

$$\bar{E}_i = E_o [\cos(45)\bar{a}_x - \sin(45)\bar{a}_z] e^{-j\frac{\pi}{3}10^4[x\sin(45)+z\cos(45)]}$$

$$\bar{E}_i = E_o \left[\frac{1}{\sqrt{2}}\bar{a}_x - \frac{1}{\sqrt{2}}\bar{a}_z \right] \times e^{-\frac{j\pi 10^4}{3\sqrt{2}}(x+z)}$$

$$\bar{E}_i = \frac{E_o}{\sqrt{2}} [\bar{a}_x - \bar{a}_z] e^{-j\frac{\pi 10^4(x+z)}{3\sqrt{2}}}$$

53. The expression for \bar{E}_r is

(A) $0.23 \frac{E_o}{\sqrt{2}} (\hat{a}_x + \hat{a}_z) e^{-j\frac{\pi \times 10^4(x-z)}{3\sqrt{2}}} \text{ V / m}$

(B) $-\frac{E_o}{\sqrt{2}} (\hat{a}_x + \hat{a}_z) e^{j\frac{\pi \times 10^4 z}{3}} \text{ V / m}$

(C) $0.44 \frac{E_o}{\sqrt{2}} (\hat{a}_x + \hat{a}_z) e^{-j\frac{\pi \times 10^4(x-z)}{3\sqrt{2}}} \text{ V / m}$

(D) $\frac{E_o}{\sqrt{2}} (\hat{a}_x + \hat{a}_z) e^{-j\frac{\pi \times 10^4(x+z)}{3}} \text{ V / m}$

Answer: (A)

Exp: (A)

The reflection co-efficient for parallel polarization is given by

$$\Gamma = \frac{E_r}{E_i} = \frac{\frac{\epsilon_{r2}}{\epsilon_{r1}} \cos \theta_i - \sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}} - \sin^2 \theta_i}}{\frac{\epsilon_{r2}}{\epsilon_{r1}} \cos \theta_i + \sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}} - \sin^2 \theta_i}}$$

$$\Gamma = \frac{\frac{4.5}{1} \cos(45) - \sqrt{\frac{4.5}{1} - \sin^2(45)}}{\frac{4.5}{1} \cos(45) + \sqrt{\frac{4.5}{1} - \sin^2(45)}}$$

$$\Gamma = 0.23$$

$$\bar{E}_r = \Gamma E_o [\cos \theta_r \bar{a}_x + \sin \theta_r \bar{a}_z] e^{-j\beta[x \sin \theta_r - z \cos \theta_r]}$$

But $\theta_r = \theta_i$ since reflected ray and incident ray lies in the same medium by using snell's law

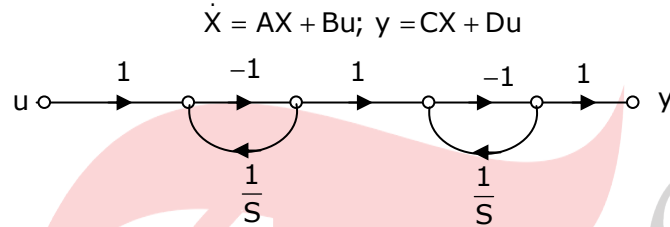
$$\bar{E}_r = \Gamma E_o [\cos \theta_i \bar{a}_x + \sin \theta_i \bar{a}_z] e^{-j\beta[x \sin \theta_i - z \cos \theta_i]}$$

$$\text{Hence } \bar{E}_r = 0.23 \times E_o [\cos(45)\bar{a}_x + \sin(45)\bar{a}_z] e^{-j\frac{\pi \times 10^4}{3} \left[x \frac{1}{\sqrt{2}} - z \frac{1}{\sqrt{2}} \right]}$$

$$E_r = 0.23 \times \frac{E_o}{\sqrt{2}} [\bar{a}_x + \bar{a}_z] e^{-j\frac{\pi \times 10^4(x-z)}{3\sqrt{2}}} \text{ V / m}$$

Statement for Linked Answer Questions: 54 & 55

The state diagram of a system is shown below. A system is described by the state-variable equations

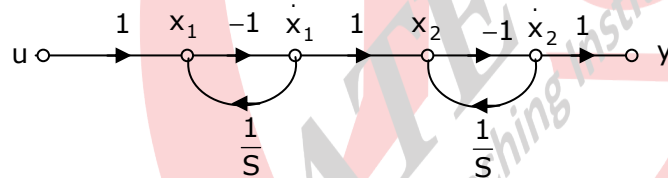


54. The state-variable equations of the system shown in the figure above are

- (A) $\dot{X} = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix} X + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u$
 $y = [1 \ -1] X + u$
- (B) $\dot{X} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix} X + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u$
 $y = [-1 \ -1] X + u$
- (C) $\dot{X} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix} X + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u$
 $y = [1 \ -1] X - u$
- (D) $\dot{X} = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} X + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u$
 $y = [-1 \ -1] X - u$

Answer: (A)

Exp:



$$\dot{x}_1 = -x_1 - u$$

$$\dot{x}_2 = -x_2 - x_1$$

$$y = -x_2 + x_1 + u$$

$$\dot{x} = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix} x + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u; y = [1 \ -1] x + u$$

55. The state transition matrix e^{At} of the system shown in the figure above is

- (A) $\begin{bmatrix} e^{-t} & 0 \\ te^{-t} & e^{-t} \end{bmatrix}$ (B) $\begin{bmatrix} e^{-t} & 0 \\ -te^{-t} & e^{-t} \end{bmatrix}$ (C) $\begin{bmatrix} e^{-t} & 0 \\ e^{-t} & e^{-t} \end{bmatrix}$ (D) $\begin{bmatrix} e^{-t} & -te^{-t} \\ 0 & e^{-t} \end{bmatrix}$

Answer: (A)

Exp: $A = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}$

$$e^{At} = L^{-1} [(SI - A)^{-1}]$$

$$e^{At} = \begin{bmatrix} e^{-t} & 0 \\ te^{-t} & e^{-t} \end{bmatrix}$$

IEC-GATE-2013 PAPER

Q. No. 56 – 60 Carry One Mark Each

56. Choose the grammatically CORRECT sentence:

- (A) Two and two add four (B) Two and two become four
(C) Two and two are four (D) Two and two make four

Answer: (D)

57. Statement: You can always give me a ring whenever you need.

Which one of the following is the best inference from the above statement?

- (A) Because I have a nice caller tune
(B) Because I have a better telephone facility
(C) Because a friend in need is a friend indeed
(D) Because you need not pay towards the telephone bills when you give me a ring

Answer: (C)

58. In the summer of 2012, in New Delhi, the mean temperature of Monday to Wednesday was 41°C and of Tuesday to Thursday was 43°C . If the temperature on Thursday was 15% higher than that of Monday, then the temperature in $^{\circ}\text{C}$ on Thursday was

- (A) 40 (B) 43 (C) 46 (D) 49

Answer: (C)

Explanations:- Let the temperature of Monday be T_M

Sum of temperatures of Tuesday and Wednesday = T and

Temperature of Thursday = T_{Th}

Now, $T_M + T = 41 \times 3 = 123$

& $T_{Th} + T = 43 \times 3 = 129$

$\therefore T_{Th} - T_M = 6$, Also $T_{Th} = 1.15T_M$

$\therefore 0.15T_M = 6 \Rightarrow T_M = 40$

\therefore Temperature of Thursday = $40 + 6 = 46^{\circ}\text{C}$

59. Complete the sentence:

Dare _____ mistakes.

- (A) commit (B) to commit (C) committed (D) committing

Answer: (B)

60. They were requested not to quarrel with others.

Which one of the following options is the closest in meaning to the word quarrel?

- (A) make out (B) call out (C) dig out (D) fall out

Answer: (D)

Q. No. 61 – 65 Carry Two Marks Each

61. A car travels 8 km in the first quarter of an hour, 6 km in the second quarter and 16km in the third quarter. The average speed of the car in km per hour over the entire journey is

- (A) 30 (B) 36 (C) 40 (D) 24

Answer: (C)

Explanations:-Average speed = $\frac{\text{Total distance}}{\text{Total time}}$

$$= \frac{8 + 6 + 16}{\frac{1}{4} + \frac{1}{4} + \frac{1}{4}} = 40 \text{ km / hr}$$

62. Find the sum to n terms of the series 10 + 84 + 734 + ...

- (A) $\frac{9(9^n + 1)}{10} + 1$ (B) $\frac{9(9^n - 1)}{8} + 1$ (C) $\frac{9(9^n - 1)}{8} + n$ (D) $\frac{9(9^n - 1)}{8} + n^2$

Answer: (D)

Explanations:-Using the answer options, substitute n = 2. The sum should add up to 94

Alternative Solution:

The given series is 10+84+734+.....+n terms

$$= (9 + 1) + (9^2 + 3) + (9^3 + 5) + (9^4 + 7) + \dots n \text{ terms}$$

$$= (9 + 9^2 + 9^3 + \dots n \text{ terms}) + (1 + 3 + 5 + 7 + \dots n \text{ terms})$$

$$= \frac{9(9^n - 1)}{9 - 1} + n^2 \left(\begin{array}{l} s_n = \frac{a(r^n - 1)}{r - 1} \quad (r > 1) \text{ and} \\ \text{Sum of first } n \text{ odd natural numbers is } n^2 \end{array} \right)$$

63. Statement: There were different streams of freedom movements in colonial India carried out by the moderates, liberals, radicals, socialists, and so on.

Which one of the following is the best inference from the above statement?

- (A) The emergence of nationalism in colonial India led to our Independence
 (B) Nationalism in India emerged in the context of colonialism
 (C) Nationalism in India is homogeneous
 (D) Nationalism in India is heterogeneous

Answer: (D)

64. The set of values of p for which the roots of the equation $3x^2 + 2x + p(p - 1) = 0$ are of opposite sign is

- (A) $(-\infty, 0)$ (B) $(0, 1)$ (C) $(1, \infty)$ (D) $(0, \infty)$

Answer: (B)

Explanation:

Since the roots are of opposite sign, the product of roots will be negative.

$$\therefore \frac{p(p - 1)}{3} < 0 \Rightarrow p(p - 1) < 0 \Rightarrow (p - 0)(p - 1) < 0 \Rightarrow 0 < p < 1$$

Thus the required set of values is $(0, 1)$

65. What is the chance that a leap year, selected at random, will contain 53 Sundays?
(A) $\frac{2}{7}$ (B) $\frac{3}{7}$ (C) $\frac{1}{7}$ (D) $\frac{5}{7}$

Answer: (A)

Explanations:-There are 52 complete weeks in a calendar year = $52 \times 7 = 364$ days

Number of days in a leap year = 366

\therefore Probability of 53 Saturdays = $\frac{2}{7}$

